EXOTIC OPTIONS

BINARY AND BARRIER OPTIONS

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Exotic options

Usually we talk about simple European or American options, or plain Vanilla options as they sometimes are called. They are standardized and exchanges or brokers on a regular basis quote their prices.

By adding additional properties or definitions to the Vanilla options, we can construct diffrent kinds of Exotic options. The Exotic options are not traded as actively as Vanilla options but they are still an important factor in a portfolio since they more profitable.
We will focus on two type of exotic options: binary options and barrier options.

1. Binary options (Digitals)

Binary options or Digitals, as they are called, provide the buyer with a Fixed payout profile. This means that the buyer receives the same payout irrespective of how far in the money the option closes. A digital option is an option whose payout is characterized as having only two potential values - a fixed payout of, say $1, when the option is in-the-money (underlying price above strike for a call and below strike for a put) or a $0 payout otherwise. The payoff remains the same, no matter how deep in-the-money the option is. There are two types of digital options: Cash-or-nothing options and Asset-or-nothing options.

1.1 Cash-or-nothing options

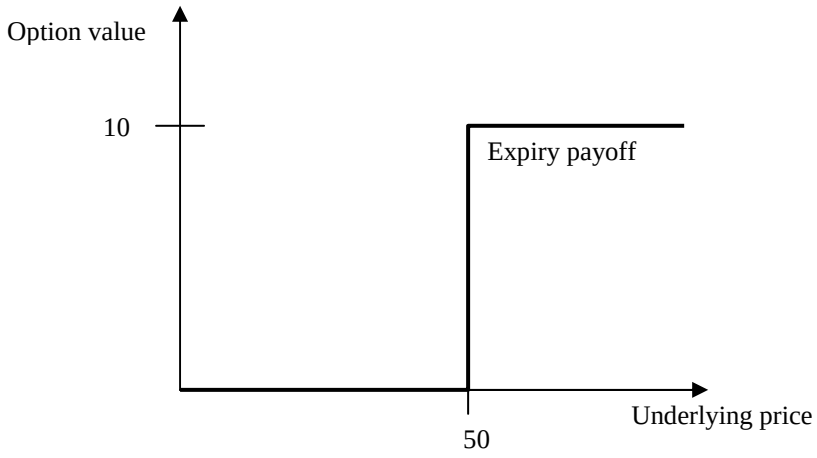
Also called all-or nothing or bet options.
They pay out a fixed amount of cash if they expire in the money, no matter how “far” it is in the money. The size of the payoff does not differ whether the option is just in the money or deep in the money. Digital options can be seen as a bet on the market reaching a certain level, with a fixed payout.

In a risk-neutral world, the probability of the asset price being above the strike price at the maturity of an option is $N(d_2)$. Therefore the value of a cash-or-noting is

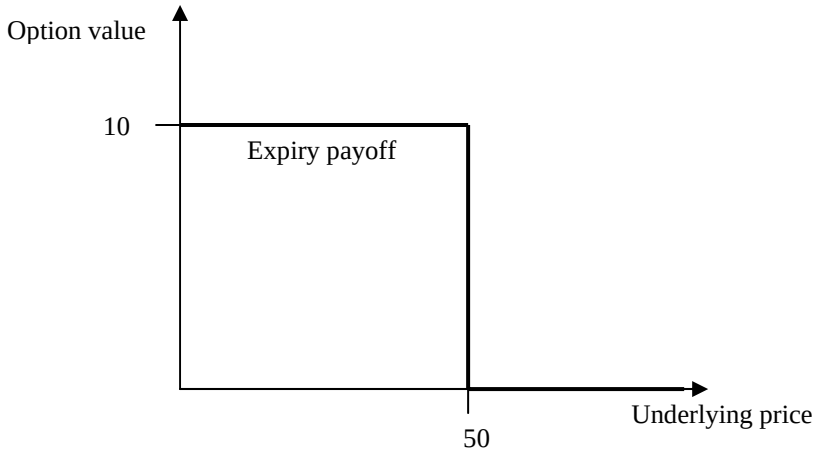
$$Qe^{-rT}N(d_2)$$

where Q is the fixed amount payout.

A payout diagram for a cash-or-nothing call with strike price of 50 and cash payout of 10



For a cash-or-nothing put with the same payout and strike the payoff would be



By using the Black & Scholes formula, we can price Cash-or-nothing options by:

$$P_{call} = Qe^{-rT}N(d)$$
$$P_{put} = Qe^{-rT}N(-d)$$

where

$$d = \frac{\ln\left(\frac{S}{K}\right) + \left(r - q + \frac{\sigma^2}{2}\right)T}{\sigma \cdot \sqrt{T}}$$

1.2 Asset-or-nothing options

The asset or nothing option gives the holder the right to buy or sell the underlying asset to a discount or premium price.
The asset-or-nothing call pays off nothing if the underlying asset price ends up below the strike price and pays an amount equal to the asset price itself if it ends up above the strike price. The asset-or-nothing put is exactly the opposite which gives us the following formulas for the value of the call- respectively put option.

$$P_{call} = S_0 e^{-qT}N(d_1)$$
$$P_{put} = S_0 e^{-qT}N(-d_1)$$

The payoff diagrams look exactly like those for the cash-or-nothing options but instead the payoff value is the asset price itself.

2. Barrier Options

A barrier option is like a plain vanilla option but with one exception: the presence of one or two price barriers. If the barrier value is touched at any time before maturity, it causes an option to come into existence (in the case of a knock-in option) or it will cause an existing option to cease to exist (in the case of a knock-out option). Barrier options are more attractive to some traders since they are less expensive than a corresponding regular option.
In this report we will bring up two types of barrier options, Knock-in and Knock-out options.

2.1 Knock-in Options

There are two kinds of knock-in options,

Up and in
Down and in

With knock-in options, the buyer starts out without a vanilla option. If the buyer has selected an upper price barrier, and the currency hits that level, it creates a vanilla option with maturity date and strike price agreed upon at the outset. This would be called an up and in. The down and in option is the same as the up and in, except the currency has to reach a lower barrier. Upon hitting the chosen lower price level, it creates a vanilla option.
The value of a down-and-in and an up-and-in call is given by,

$$c_{di} = S_0 e^{-qT} (H/S_0)^{2\lambda} N(y) - K e^{-rT} (H/S_0)^{2\lambda-2} N(y - \sigma\sqrt{T})$$

where

$$\lambda = \frac{r - q + \sigma^2/2}{\sigma^2}$$

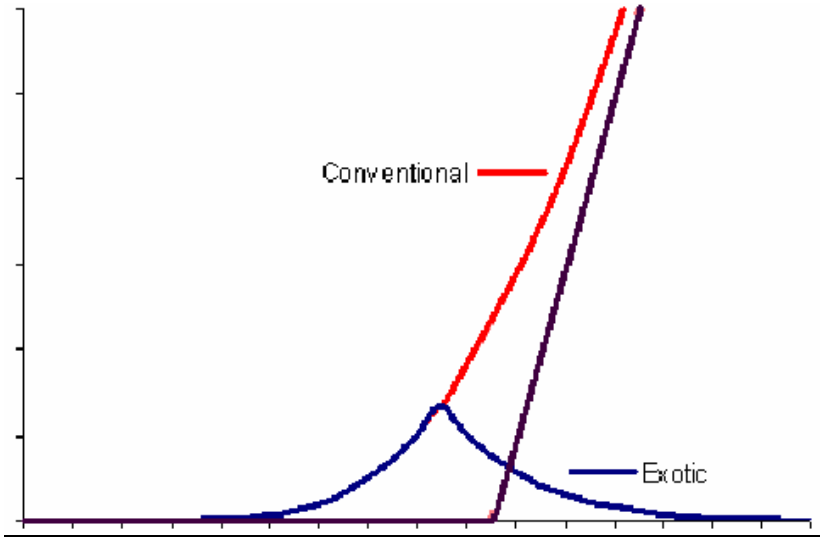
$$y = \frac{\ln\left[H^2/(S_0 K)\right]}{\sigma\sqrt{T}} + \lambda\sigma\sqrt{T}$$

and

$$c_{ui} = S_0 N(x_1) e^{-qT} - Ke^{-rT} N(x_1 - \sigma\sqrt{T}) - S_0 e^{-qT} (H/S_0)^{2\lambda} \left[N(-y) - N(-y_1)\right]$$
$$+ Ke^{-rT} (H/S_0)^{2\lambda-2} \left[N(-y + \sigma\sqrt{T}) - N(-y_1 + \sigma\sqrt{T})\right]$$

The payoff of a down-and-in call is shown below,

Down-and-in call



2.2 Knock-out Options

These options are the reverse of knock-ins. As in the knock-in option, there are two kinds of knock-out options,

Up and out.

Down and out

With knock-outs, the buyer begins with a vanilla option, however, if the predetermined price barrier is hit, the vanilla option is cancelled and the seller has no further obligation. If the option hits the upper barrier, the option is cancelled and you lose your premium paid, thus, "up and out". If the option hits the lower price barrier, the option is cancelled, thus, "down and out".

The value of a down-and-out and an up-and-out call is just a regular call option minus its corresponding knock-in option:

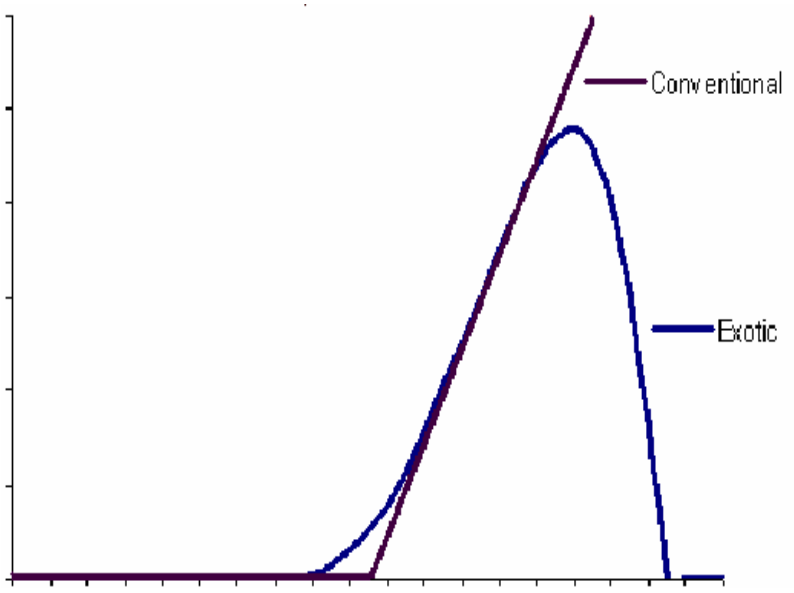
$$c_{do} = c - c_{di}$$

and

$$c_{uo} = c - c_{ui}$$

An example of a payoff for a up-and-out option is the following graph.

Up-and-Out Call



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